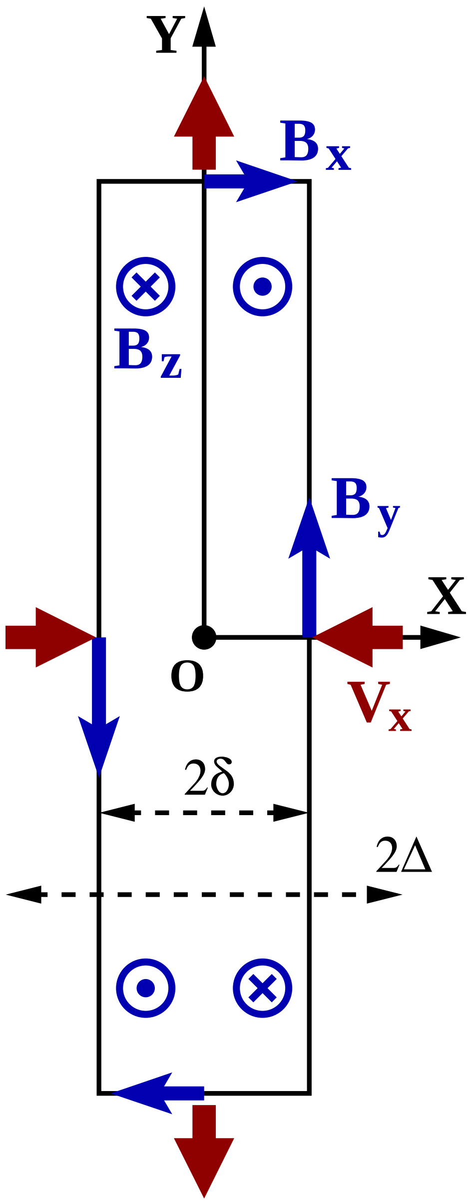
Hall reconnection and beyond

Leonid Malyshkin

(University of Chicago, CMSO)

CMSO meeting, Santa Fe, Apr 2009

The problem setup & assumptions



We assume

- non-relativistic incompressible two-component plasma that is composed of p^+ and e^-;
- isotropic p^+ and e^- pressure
- η is constant and small, large Lundquist number;
- neglect viscosity;
- 2-dimensional geometry, $\partial/\partial z \equiv 0$;
- geometric symmetries of the layer;
- quasi-stationarity, $\partial/\partial t \approx 0$:
- quadrupole B_z;

- **for simplicity of presentation,** assume $\mathbf{u}^p \ll \mathbf{u}^e$ or $\mathbf{j} \gg ne\mathbf{V}$.

Equations

- Equations of the motion of the e^- and p^+:

$$nm_e[\partial_t \mathbf{u}^e + (\mathbf{u}^e \nabla)\mathbf{u}^e] = -\nabla P_e - ne(\mathbf{E} + \mathbf{u}^e \times \mathbf{B}) + ne\eta \mathbf{j},$$

$$nm_p[\partial_t \mathbf{u}^p + (\mathbf{u}^p \nabla)\mathbf{u}^p] = -\nabla P_p + ne(\mathbf{E} + \mathbf{u}^p \times \mathbf{B}) - ne\eta \mathbf{j}.$$

- Express $\mathbf{u}^e$ and $\mathbf{u}^p$ in terms of $\mathbf{j}$ and $\mathbf{V}$:

$$\mathbf{u}^p = \mathbf{V} \quad \text{and} \quad \mathbf{u}^e = \mathbf{V} - \frac{1}{ne}\mathbf{j} \approx -\frac{1}{ne}\mathbf{j} \qquad \text{because} \quad m_e \ll m_p.$$

- Obtain generalized Ohm's law from the equation of e^- motion

$$ne\mathbf{E} = ne\eta \mathbf{j} + \mathbf{j} \times \mathbf{B} - \nabla P_e + ned_e^2 \partial_t \mathbf{j} - d_e^2 (\mathbf{j}\nabla)\mathbf{j}.$$

- Obtain momentum equation from the equation of p^+ motion

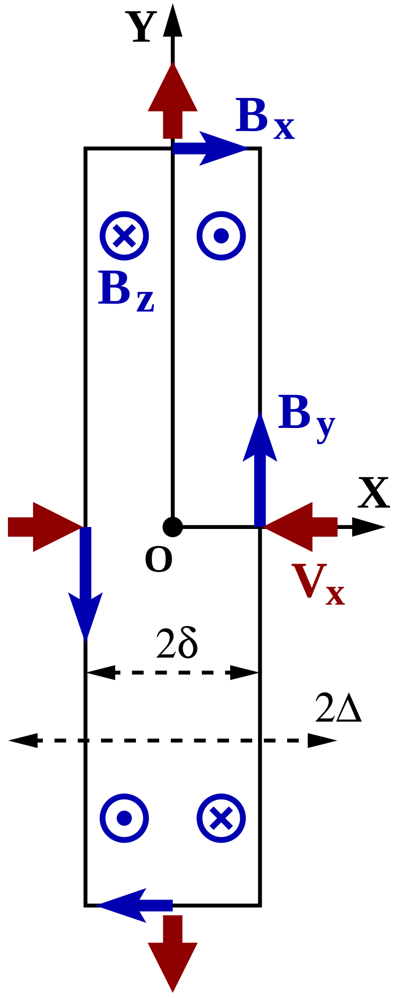
$$nm_p\,[\partial_t \mathbf{V} + (\mathbf{V}\nabla)\mathbf{V}] = -\nabla P + \mathbf{j} \times \mathbf{B} - d_e^2(\mathbf{j}\nabla)\mathbf{j}.$$

- Maxwell equations

Equations (continue)

Ampere's Law z-component (at O-point):

$$j_o = (j_z)_o \approx \frac{B_{ext}}{\delta}$$



Equations (continue)

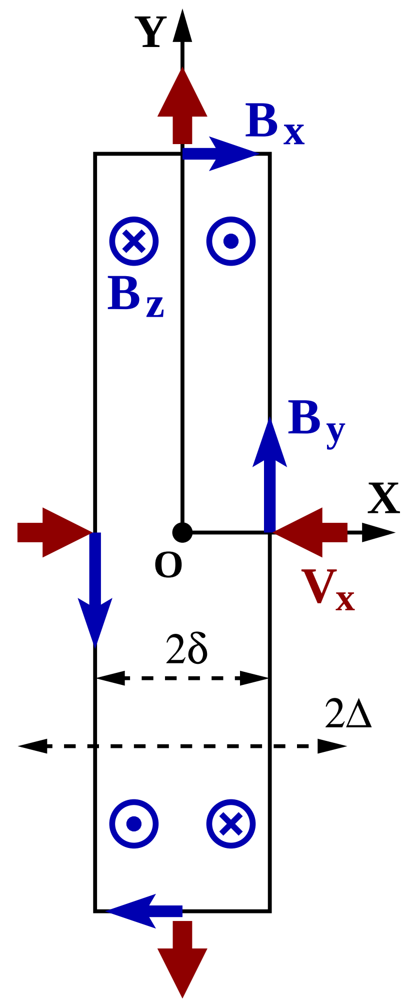
x-component of the momentum equation (force balance across the layer):

$$(\partial^2_{yy} P)_o \approx (\partial^2_{yy} B_y^2/2)_{ext} = -2B^2_{ext}/L^2 < 0$$

y-component of the momentum equation
(acceleration along the layer):

at the O-point, calculate $\partial/\partial y$ of

$$nm_p(\mathbf{V}\nabla)V_y = -\partial_y P + j_z B_x - j_x B_z - d_e^2(\mathbf{j}\nabla)j_y$$



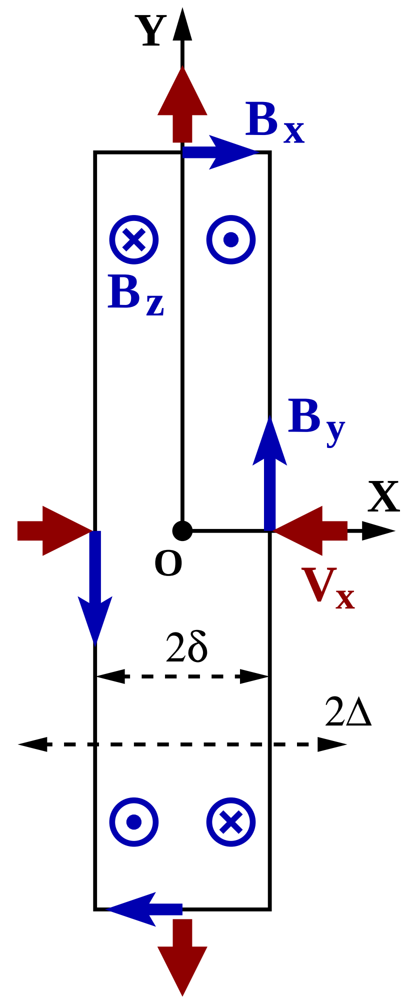
Equations (continue)

Faraday's Law x- and y-components:

$$(\nabla \times \mathbf{E})_x = \partial_y E_z = -\partial B_x / \partial t \approx 0,$$

$$(\nabla \times \mathbf{E})_y = -\partial_x E_z = -\partial B_y / \partial t \approx 0,$$

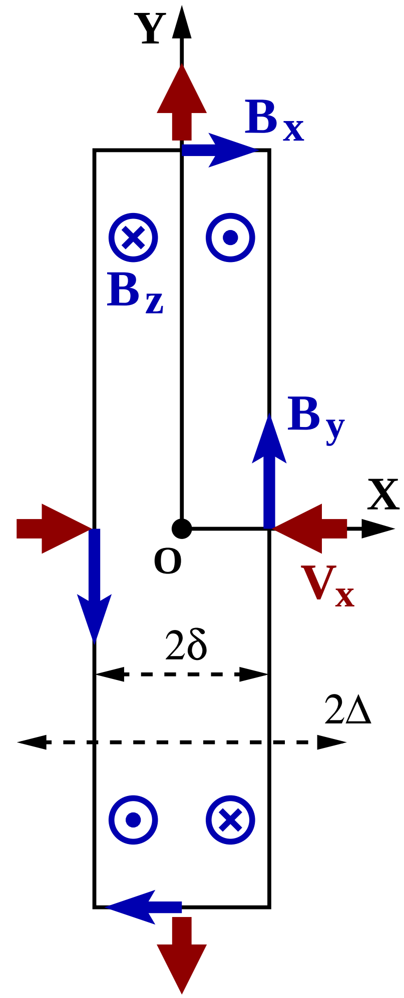
$\Rightarrow$ $E_z \approx$ constant in space



Equations (continue)

Ohm's Law z-component:

$$neE_z = ne\eta j_z + j_x B_y - j_y B_x - d_e^2(\mathbf{j}\nabla)j_z$$
$$= \text{constant}$$



- O-point:

$$E_z = \eta j_o$$

- $E_z \approx$ const across the layer:

$$\boxed{\frac{\partial^2}{\partial x^2} E_z \approx 0 \quad \text{at the point O}}$$

- $E_z \approx$ const along the layer:

$$\boxed{\frac{\partial^2}{\partial y^2} E_z \approx 0 \quad \text{at the point O}}$$

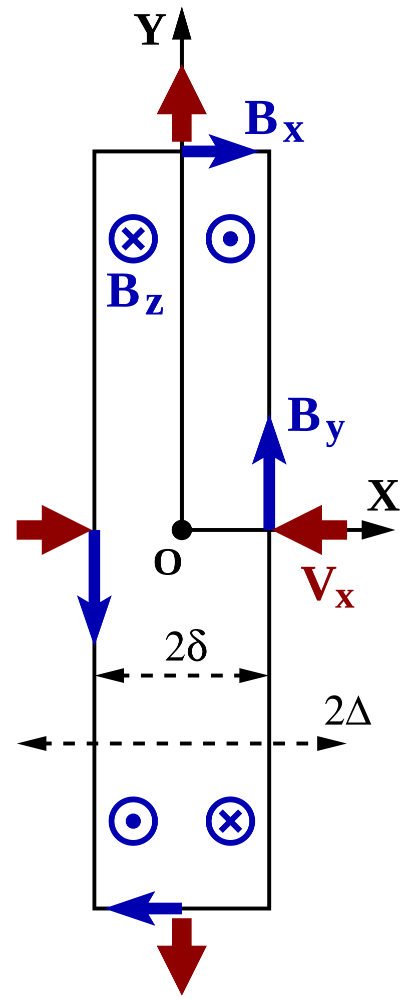
Equations (continue)

Faraday's Law z-component:

$$\partial_x E_y - \partial_y E_x = -\partial B_z / \partial t \approx 0$$

at the O-point, calculate $\partial^2/\partial x \partial y$ of

$$\partial_x E_y - \partial_y E_x \approx 0$$



Equations (summary)

Ampere's Law (z-): 1 equation

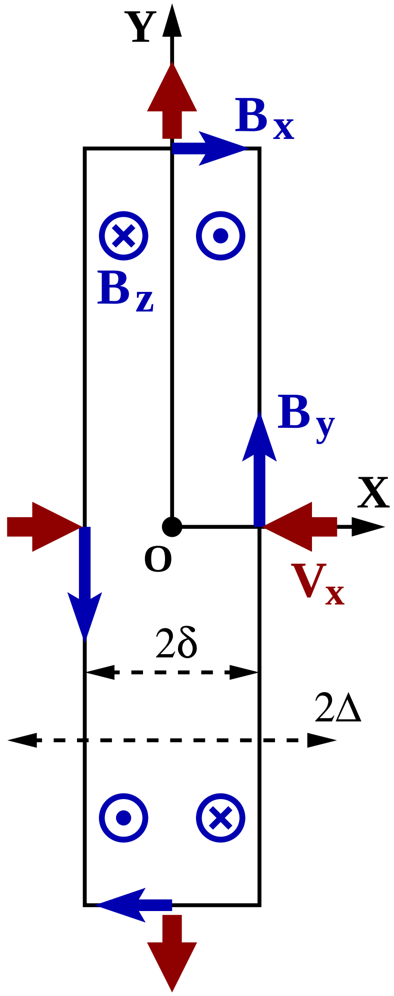
Momentum Equation (y-): 1 equation

Faraday's and Ohm's Laws (x-, y-, z-): 3 equations

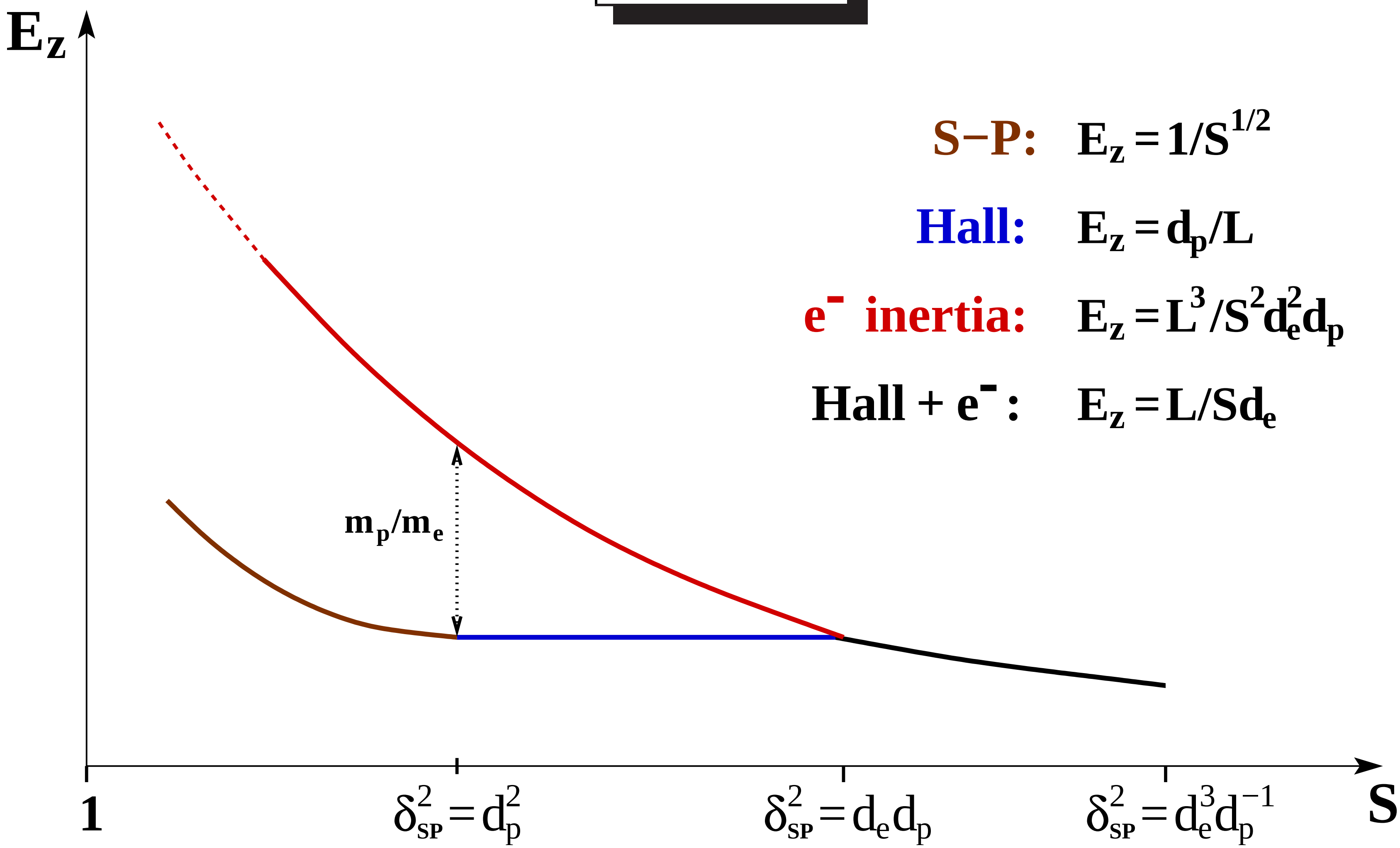
5 unknowns: j_o, δ, $(\partial_y V_y)_o$, $(\partial_y B_x)_o$ and $(\partial^2_{xy} B_z)_o$

Find reconnection rate: $E_z = \eta j_o$

Introduce $V_A = \frac{B_{ext}}{\sqrt{\rho}}$, $S = \frac{L V_A}{\eta}$, $d_p = \frac{m_p}{e\sqrt{\rho}}$, $d_e = \frac{m_e}{e\sqrt{\rho}}$



Solution



e^- acceleration: $d_e^2 j_y \partial_y j_y = -neE_y - \partial_y P_e + ne\eta j_y + j_o B_x.$

p^+ acceleration: $nm_p V_y \partial_y V_y = -\partial_y P + j_z B_x - d_e^2 j_y \partial_y j_y.$

Summary

- The central result is that when electron inertia is included, there are two branches of solutions for the reconnection rate – one slow and one fast.
- The slow rate is slow sub-Alfvenic, $E_z^{slow} \ll V_A B_{ext}$ (assuming $S \gg 1$ and $d_p \ll L$). The slow reconnection regime may correspond to "quiet" periods of gradual accumulation of magnetic energy in an (astro)physical system.
- The fast rate can be comparable to the Alfvenic rate $V_A B_{ext}$ if η is enhanced over Spitzer value by kinetic effects in the plasma. The fast reconnection regime can correspond to a rapid release of the accumulated magnetic energy.