Fast and slow magnetic reconnection

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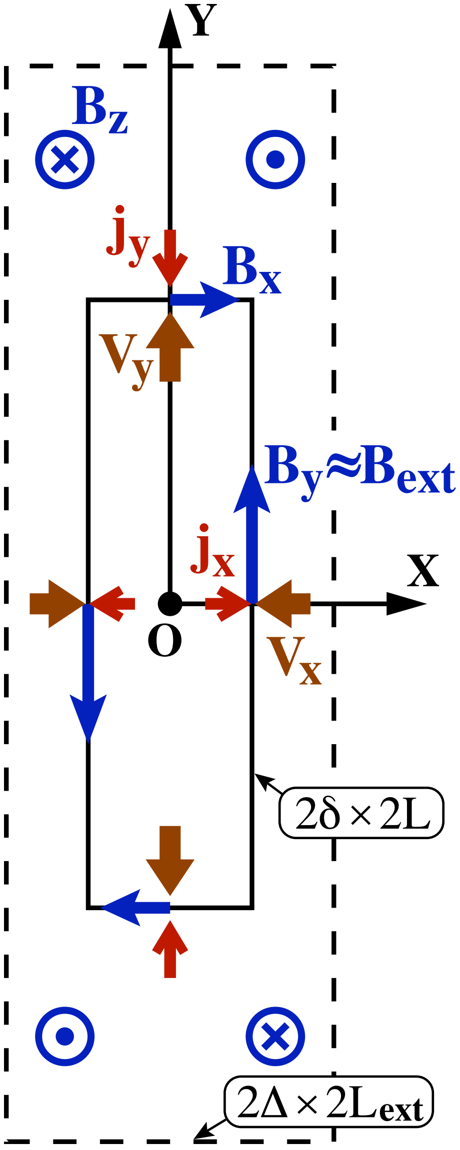
The problem setup & assumptions



Assumptions:

- non-relativistic incompressible p^+ and e^- plasma;
- isotropic p^+ and e^- pressure, also neglect viscosity;
- resistivity η is constant & small (large Lundquist);
- a thin layer, 2-dimensional geometry $\partial/\partial z \equiv 0$;
- geometric symmetries of the layer, quadrupole B_z;
- quasi-stationarity, $\partial/\partial t \approx 0$.

Notations:

- we use physical units in which $c = 1$ and $4\pi = 1$;
- 2δ and $2L$ are thicknesses and length of e^- layer;
- 2Δ and $2L_{ext}$ are thicknesses & length of p^+ layer;
- B_{ext} is upstream field (B_y at $y=0$ and $x \approx \Delta$);
- $V_A = B_{ext}/\sqrt{\rho}$ is Alfven velocity, $\rho = m_p n = \text{const}$;
- $S = L_{ext} V_A/\eta$ is Lundquist number;
- $d_p = c/\omega_{pp} = \sqrt{m_p}/e\sqrt{n}$ and $d_e = c/\omega_{pe} = \sqrt{m_e}/e\sqrt{n}$ are the proton and electron inertial lengths.

Equations

- Equations of the motion of the e^- and p^+:

$$nm_e[\partial_t \mathbf{u}^e + (\mathbf{u}^e\nabla)\mathbf{u}^e] = -\nabla P_e - ne(\mathbf{E} + \mathbf{u}^e \times \mathbf{B}) + ne\eta\mathbf{j},$$
$$nm_p[\partial_t \mathbf{u}^p + (\mathbf{u}^p\nabla)\mathbf{u}^p] = -\nabla P_p + ne(\mathbf{E} + \mathbf{u}^p \times \mathbf{B}) - ne\eta\mathbf{j}.$$

- Express $\mathbf{u}^e$ and $\mathbf{u}^p$ in terms of electric current $\mathbf{j} = ne(\mathbf{u}^p - \mathbf{u}^e)$ and center-of-mass velocity $\mathbf{V} = (m_p\mathbf{u}^p + m_e\mathbf{u}^e)/\rho$, also use $m_e \ll m_p$:

$$\mathbf{u}^p = \mathbf{V} + (m_e/m_p)\,\mathbf{j}/ne \quad \text{and} \quad \mathbf{u}^e = \mathbf{V} - \mathbf{j}/ne.$$

- Obtain generalized Ohm's law from the equation of e^- motion

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{j} + \mathbf{j}\times\mathbf{B}/ne - (1/ne)\left[\nabla P_e - (d_e^2/d_p^2)\nabla P_p\right] -$$
$$+\, d_e^2\left[\partial_t \mathbf{j} + (\mathbf{V}\nabla)\mathbf{j} + (\mathbf{j}\nabla)\mathbf{V} - (1/ne)(\mathbf{j}\nabla)\mathbf{j}\right].$$

- Obtain momentum equation from the equation of p^+ motion

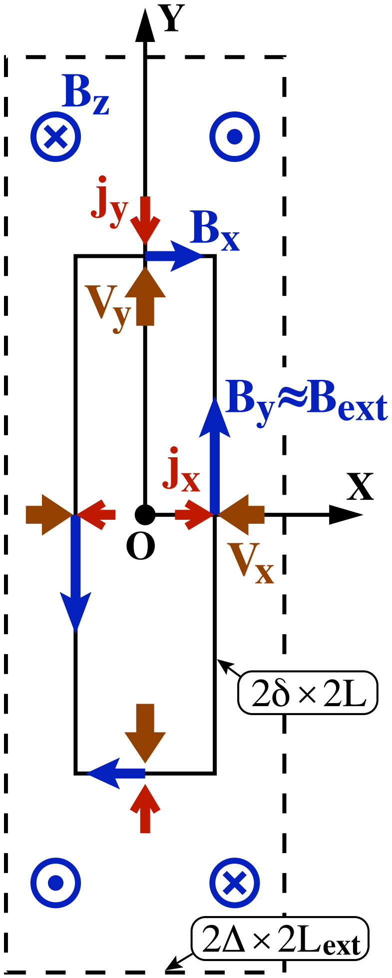
$$nm_p\,[\partial_t \mathbf{V} + (\mathbf{V}\nabla)\mathbf{V}] + d_e^2(\mathbf{j}\nabla)\mathbf{j} = -\nabla P + \mathbf{j}\times\mathbf{B}, \qquad P = P_e + P_p.$$

- Maxwell equations: $\nabla\times\mathbf{B} = \mathbf{j}, \quad -\partial_t\mathbf{B} = \nabla\times\mathbf{E}, \quad \nabla\cdot\mathbf{B} = 0.$

Equations (continue)

Ampere's Law z-component (at O-point):

$$\boxed{j_o = (j_z)_o \approx \frac{B_{ext}}{\delta}}$$



Equations (continue)

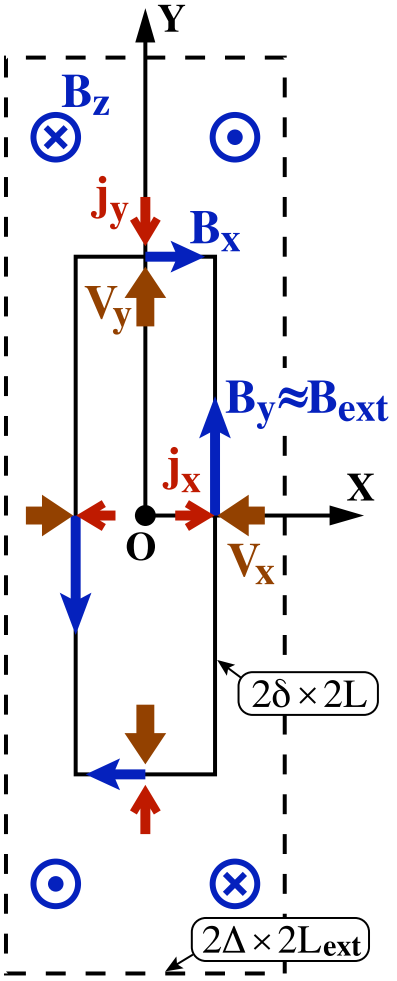
x-component of the momentum equation (force balance across the layer):

$$(\partial_{yy} P)_o \approx (\partial_{yy} B_y^2/2)_{ext} \approx -2B_{ext}^2/L^2$$

y-component of the momentum equation
(acceleration along the layer):

at the O-point, calculate $\partial/\partial y$ of

$$nm_p(\mathbf{V}\nabla)V_y + d_e^2(\mathbf{j}\nabla)j_y = -\partial_y P + j_z B_x - j_x B_z$$



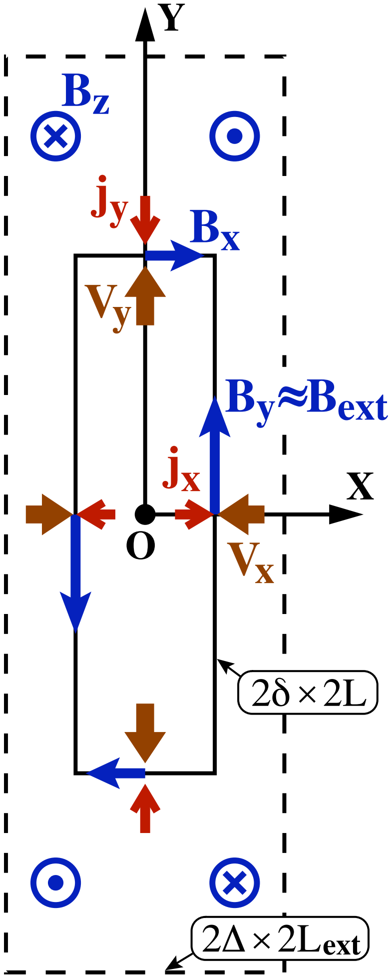
Equations (continue)

Faraday's Law x- and y-components:

$$(\nabla \times \mathbf{E})_x = \partial_y E_z = -\partial B_x/\partial t = 0,$$

$$(\nabla \times \mathbf{E})_y = -\partial_x E_z = -\partial B_y/\partial t = 0,$$

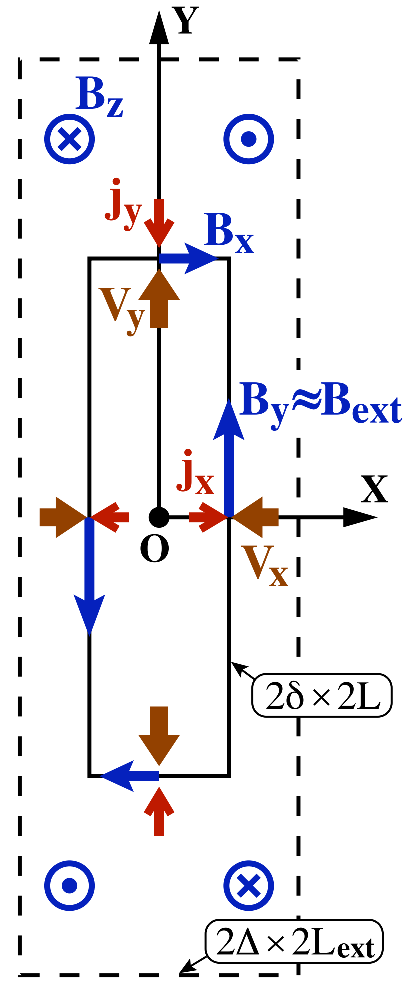
$\Rightarrow$ E_z = constant in space



Equations (continue)

Ohm's Law z-component:

$$\begin{aligned} E_z &= \eta j_z - V_x B_y + V_y B_x + (j_x B_y - j_y B_x)/ne \\ &\quad + d_e^2 \big[V_x \partial_x j_z + V_y \partial_y j_z + j_x \partial_x V_z + j_y \partial_y V_z \\ &\qquad - (j_x \partial_x j_z + j_y \partial_y j_z)/ne \big] = \text{constant.} \end{aligned}$$



- O-point: $E_z = \eta j_o$
- E_z = const across the layer:

$$\boxed{\partial_{xx} E_z = 0 \quad \text{at point O;}}$$

$$\boxed{\eta j_o \; = \; E_z(x{\approx}\delta, y{=}0) \; = \; E_z(x{\approx}\Delta, y{=}0)}$$

- E_z = const along the layer:

$$\boxed{\partial_{yy} E_z = 0 \quad \text{at point O;}}$$

$$\boxed{\eta j_o \; = \; E_z(x{=}0, y{\approx}L)}$$

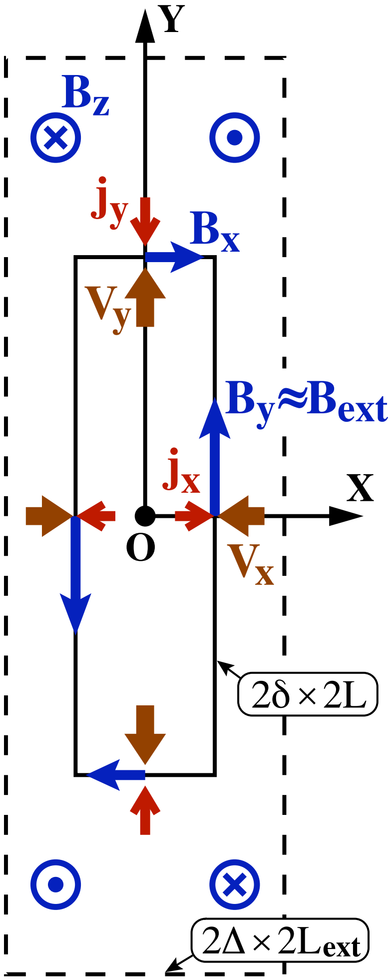
Equations (continue)

Faraday's Law z-component:

$$\partial_x E_y - \partial_y E_x = -\partial B_z/\partial t = 0$$

at the O-point, calculate $\partial^2/\partial x \partial y$ of

$$\partial_x E_y - \partial_y E_x = 0$$



Equations (summary)

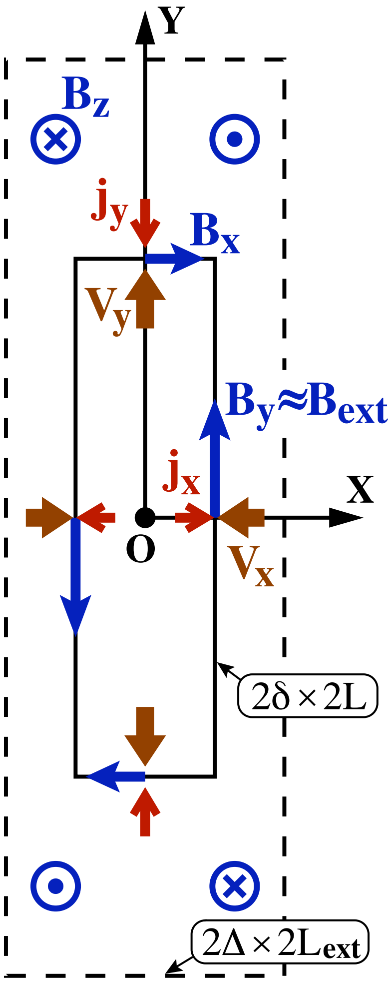
Ampere's Law (z-): 1 equation

Momentum Equation (y-): 1 equation

Faraday's and Ohm's Laws (x-, y-, z-): 6 equations

7 unknowns: j_o, δ, Δ, L, $(\partial_y V_y)_o$, $(\partial_y B_x)_o$, $(\partial_{xy} B_z)_o$
(equations partly replicate each other)

Find the unknowns and reconnection rate $E_z = \eta j_o$



Solution

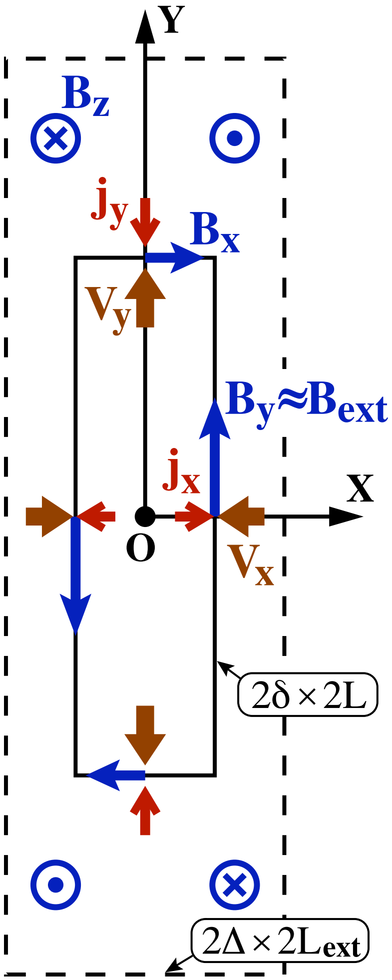
Convenient physical parameters:

$$V_A = \frac{B_{ext}}{\sqrt{nm_p}}$$ is Alfven velocity

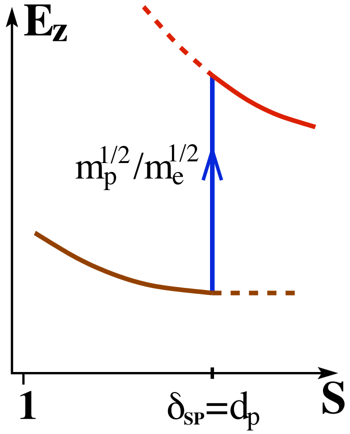
$$S = \frac{V_A L_{ext}}{\eta}$$ is Lundquist number

Convenient dimensionless parameter:

$$\tilde{\gamma} \equiv \frac{V_A d_p (\partial_{xy} B_z)_o}{B_{ext}(\partial_y V_y)_o} \approx \frac{(\mathbf{j} \times \mathbf{B})_z / ne}{(-\mathbf{V} \times \mathbf{B})_z}$$



Solution: Sweet-Parker



Solution for $\tilde{\gamma} \lesssim 1$, Sweet-Parker reconnection regime

$$S \lesssim L_{ext}^2/d_p^2 \quad \Rightarrow \quad \delta \approx \Delta \gtrsim d_p$$

$$j_o \approx \frac{\sqrt{S}\,B_{ext}}{L_{ext}}, \qquad E_z \approx \frac{1}{\sqrt{S}} V_A B_{ext}$$

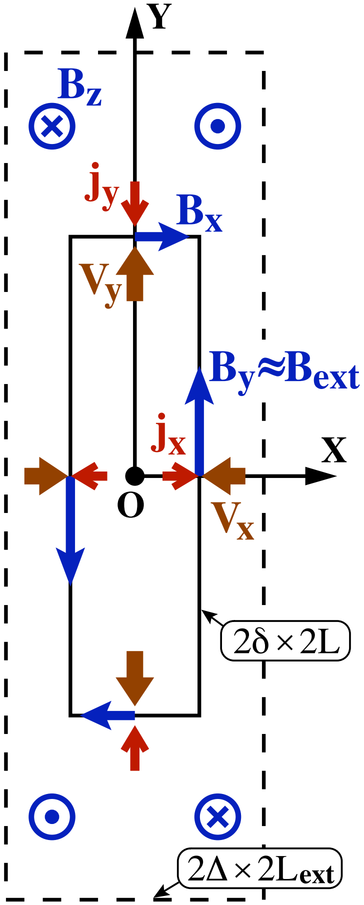
$$\delta \approx \Delta \approx \frac{L_{ext}}{\sqrt{S}}$$

$$L \approx L_{ext}$$

$$\tilde{\gamma} \approx \frac{S d_p^2}{L_{ext}^2}$$

$$(\partial_y V_y)_o = \frac{V_A}{L_{ext}}, \qquad V_y \approx V_A$$

$$(\partial_y B_x)_o \approx \frac{B_{ext}}{\sqrt{S}\,L_{ext}}, \qquad (\partial_{xy} B_z)_o \approx \frac{S B_{ext} d_p}{L_{ext}^3}$$



Solution: Hall

Solution for $1 \lesssim \tilde{\gamma} \lesssim d_p/d_e$, Hall reconnection regime

$$S \approx L_{ext}^2/d_p^2$$

$$L_{ext} \gtrsim L \gtrsim \frac{d_e L_{ext}}{d_p}, \qquad j_o \approx \frac{L_{ext} B_{ext}}{d_p L}$$

$$E_z \approx \frac{d_p}{L} V_A B_{ext} \quad \text{(S. W. H. Cowley, 1985)}$$

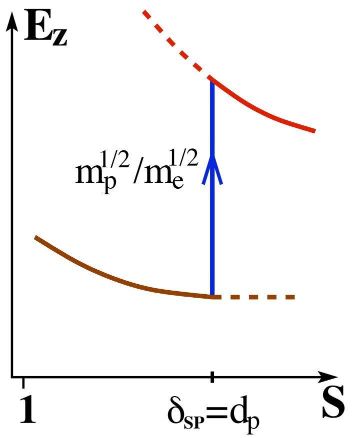
$$\delta \approx \frac{d_p L}{L_{ext}}, \qquad \Delta \approx d_p$$

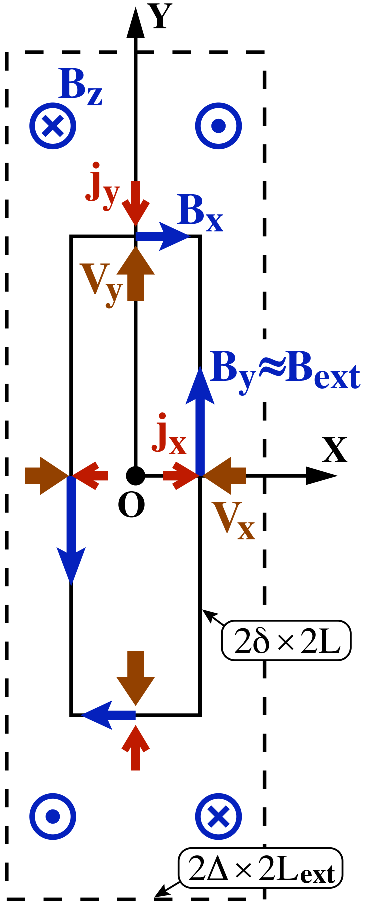
$$\tilde{\gamma} \approx \frac{L_{ext}}{L}$$

$$(\partial_y V_y)_o = \frac{V_A}{L}, \qquad V_y \approx V_A$$

$$(\partial_y B_x)_o \approx \frac{d_p B_{ext}}{L_{ext} L}, \qquad (\partial_{xy} B_z)_o \approx \frac{B_{ext} L_{ext}}{d_p L^2}$$

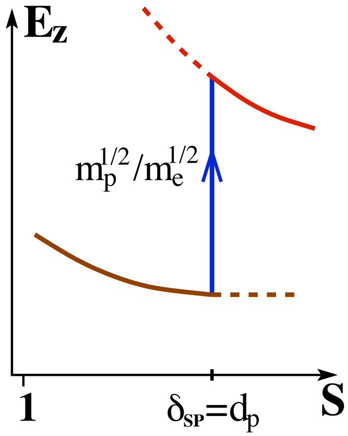
Note: $B_z \approx (\partial_{xy} B_z)_o \, \delta \, L \approx B_{ext}$





Solution: e-inertia

Solution for $d_p/d_e \lesssim \tilde{\gamma} < d_p^2/d_e^2$, e-inertia regime



$$j_o \approx \frac{B_{ext}}{d_e}$$

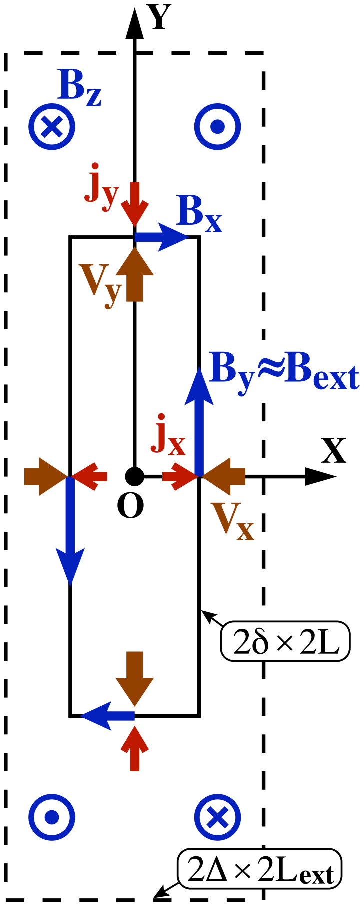
$$E_z \approx \frac{L_{ext}}{Sd_e} V_A B_{ext} \approx \frac{d_p}{L} V_A B_{ext} \qquad \text{(Zocco } et\ al.\text{, 2008)}$$

$$L \approx \frac{Sd_e d_p}{L_{ext}}, \qquad \delta \approx d_e, \qquad \Delta \approx d_p$$

$$\frac{d_p}{d_e} \lesssim \tilde{\gamma} < \frac{d_p^2}{d_e^2}$$

$$(\partial_y V_y)_o = \frac{L_{ext} V_A}{S d_e^2 \tilde{\gamma}}, \qquad (\partial_y V_y)_{x>\delta} \approx \frac{V_A}{L}$$

$$(\partial_y B_x)_o \approx \frac{L_{ext}^2 B_{ext}}{S^2 d_e d_p^2}, \qquad (\partial_{xy} B_z)_o \approx \frac{B_{ext} L_{ext}}{S d_e^2 d_p}$$



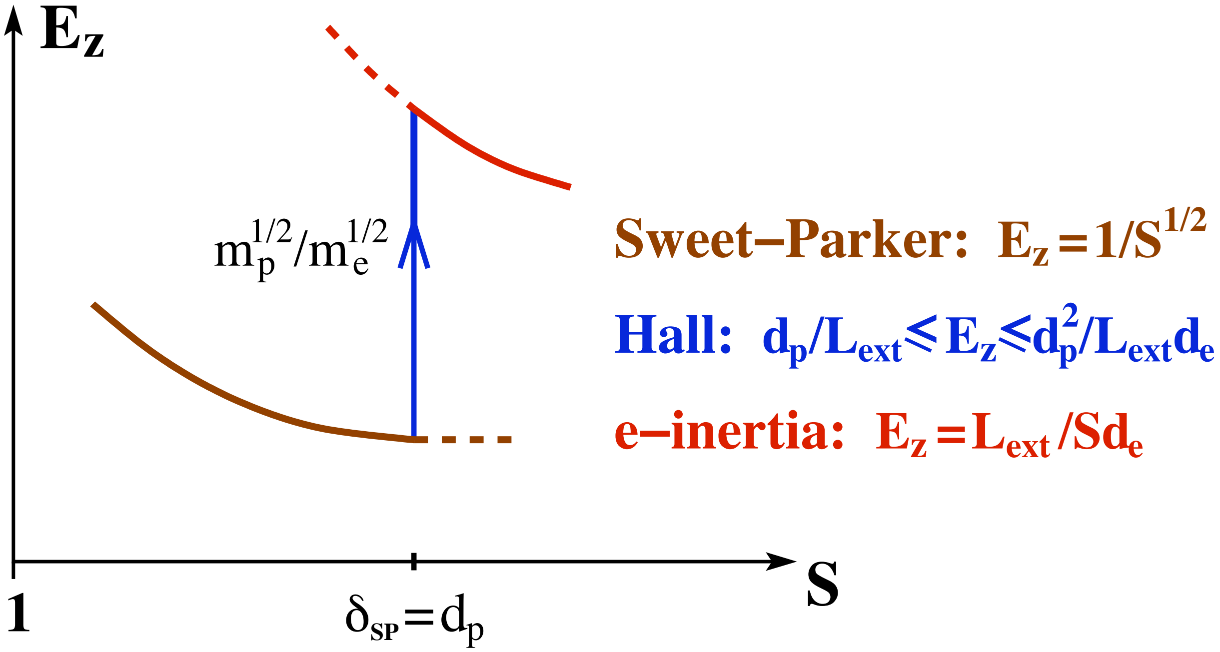
Note: $B_z \approx (\partial_{xy} B_z)_o \, \delta \, L \approx B_{ext}$, $V_y(y \approx L_{ext}) \approx V_A$,
$u_y^e \approx V_{Ae} = B_{ext}/\sqrt{n m_e}$

Summary

- e^- inertia term $d_e^2(\mathbf{j}\nabla)\mathbf{j} \approx m_e n(\mathbf{u}^e\nabla)\mathbf{u}^e$ enters the momentum equation:

$$m_p n\left[\partial_t \mathbf{V} + (\mathbf{V}\nabla)\mathbf{V}\right] + d_e^2(\mathbf{j}\nabla)\mathbf{j} \; = \; -\nabla P + \mathbf{j}\times\mathbf{B}$$

- There are slow Sweet-Parker and fast e-inertia-dominated regimes:



- http://arxiv.org/abs/0904.0660
- Need to account for anisotropy of the electron pressure tensor.